

American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2005-2006

quiz # 2

Name:

ID #:

1. (25 points) Let $f(x, y) = \ln(4 - x^2 - y^2)$.

- a. what are the domain D_f and the range R of f ?
- b. what is the boundary of D_f ? is the domain closed, open, bounded? Justify.
- c. find the equation of the level curve that passes through the point $(\sqrt{2}, 1)$.
- d. what are the level curves of f ?

2. (20 points) The Fourier series expansion of the function $f(x) = \begin{cases} 0 & -\pi < x \leq 0 \\ x & 0 \leq x < \pi \end{cases}$ is

$$\frac{\pi}{4} + \sum_{n=1}^{+\infty} \frac{1}{\pi} \left(\frac{(-1)^n - 1}{n^2} \right) \cos(nx) + \sum_{n=1}^{+\infty} b_n \sin(nx)$$

a. find b_n

b. use the series in part a) to find $\sum_{k=0}^{+\infty} \frac{1}{(2k+1)^2}$.

3. (30 points) Let (C) the curve of equation $r = \cos(2\theta)$.

- a. show that the x -axis and the y -axis are axis of symmetry for (C) .
- b. graph the curve (C) .
- c. find the area inside one of the leaves of (C) .

4. (10 points) Find $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - y^2}{\sqrt{x} - \sqrt{y}}$

5. (15 points) Use the two path test to show that $f(x, y) = \frac{\cos(x+y) - 1}{x^2 + y^2}$ does not have a limit at $(0, 0)$.

6. (bonus: 5 points) Let $f(x, y) = \begin{cases} x^2 e^{-\frac{1+y^2}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Find $\frac{\partial f}{\partial x}(0, 0)$.